International Conference on Einstein and Harmonic Manifolds

- Dates: June 19-22, 2023
- Venue: 31351-A,
  National Sciences Campus,
  Sungkyunkwan University,
  Suwon, Korea

Invited Speakers:

Balázs Csikós (Eötvös Loránd University, Hungary)
Gerhard Knieper (Ruhr-Universität Bochum, Germany)
Ramiro Lafuente (University of Queensland, Australia)
Paul-Andi Nagy (Institute for Basic Science, Korea)
Norbert Peyerimhoff (Durham University, England)
Hemangi Shah (Harish-Chandra Research Institute, India)
Joseph A. Wolf (University of California, Berkeley, USA)

Organizers:

Gye-Seon Lee (Seoul National University, Korea)
Yuri Nikolayevsky (La Trobe University, Australia)
JeongHyeong Park (Sungkyunkwan University, Korea)

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Timetable

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Abstracts

June 19, Monday

A condition for a Riemannian manifold to be globally homogeneous -Part 1-

10:00 am

Joseph A. Wolf
University of California, Berkeley, USA

Consider a connected homogeneous Riemannian manifold (M, ds^2) and a Riemannian covering $(M, ds^2) \to \Gamma\backslash(M, ds^2)$. If $\Gamma\backslash(M, ds^2)$ is homogeneous then every $\gamma \in \Gamma$ is an isometry of constant displacement. The Homogeneity Conjecture suggests the converse: if every $\gamma \in \Gamma$ is an isometry of constant displacement on (M, ds^2) then $\Gamma\backslash(M, ds^2)$ is homogeneous. We survey the cases in which the Homogeneity Conjecture has been verified, plus some new results, and suggest some related open problems.

First hour: Background, spaces of constant curvature, Riemannian symmetric spaces

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New developments on Einstein manifolds with symmetry -Part 1-

11:00 am

Ramiro Lafuente
University of Queensland, Australia

I will discuss the recent resolution of the 1975 Alekseevskii conjecture on homogeneous Einstein manifolds, in joint work with Christoph Böhm. I will also describe new results on the structure and rigidity of Einstein manifolds with a cocompact symmetry group, including the fact that a negative Einstein manifold with a cocompact unimodular group of isometries must split isometrically as a product of a compact Einstein manifold and a symmetric space of non-compact type.

2:00 pm

Non-compact harmonic manifolds and hyperbolicity

Gerhard Knieper
Ruhr-Universität Bochum, Germany

In this talk we will provide a survey on the geometry of simply connected, non-compact harmonic spaces focusing on various features of hyperbolicity. It turns out that for such spaces X the following properties are equivalent:

- X has geometric rank 1.
- X has purely exponential volume growth
- X is Gromov hyperbolic
- The geodesic flow on X is Anosov with respect to the Sasaki metric

We will show that non-flat harmonic manifolds of non-positive curvature (or more generally no focal points) imply the above properties.

In particular this includes the Damek-Ricci spaces. Combining those results with rigidity theorems of Besson, Courtois and Gallot as well as Benoist, Foulon and Labourie we obtain that all non-compact and non-flat harmonic manifolds admitting a compact quotient are rank 1 symmetric spaces of non-compact type, provided the metric has no focal points or the fundamental of the quotient is Gromov hyperbolic.

Some of the results have been generalized in collaboration with Norbert Peyerimhoff to asymptotically harmonic manifolds which we briefly mention if time permits.

3:00 pm

Some geometric characterizations of harmonic and D'Atri spaces

Balázs Csikós
Eötvös Loránd University, Hungary

Some geometric characterizations of harmonic and D'Atri spaces Abstract: First we review some geometric characterizations of harmonic spaces in terms of the volume of the intersection of two geodesic balls, or the volume, the total mean curvature, or the total scalar curvature of tubular hypersurfaces about regularly parameterized curves. One of the characterizations says that if a connected Riemannian manifold of dimension at least 4 is harmonic, then the total scalar curvatures of tubes of small radius about an arbitrary regular curve depend only on the length of the curve and the radius of the tube, and conversely, if the latter condition holds for cylinders, i.e., for tubes about geodesic segments, then the manifold is harmonic. We show that in contrast to the higher dimensional case, a connected 3-dimensional Riemannian manifold has the above mentioned property of tubes if and only if the manifold is a D'Atri space, furthermore, if the space has bounded sectional curvature, then it is enough to require the total scalar curvature condition just for cylinders to imply that the space is D'Atri. This result gives a negative answer to a question posed by L. Gheysens and L. Vanhecke. To prove the latter statements, we give a characterization of D'Atri spaces in terms of the total scalar curvature of geodesic hemispheres in any dimension.

Some analytic characterizations of harmonic manifolds

4:30 pm

JeongHyeong Park
Sungkyunkwan University, Korea

A Riemannian manifold is called harmonic if there exists a non-constant radial harmonic function in a punctured neighborhood for any point. There are many other characterizations of harmonic spaces. In this talk, we characterize harmonic manifolds in terms of the radial eigenspaces of the Laplacian. We construct examples of centrally harmonic manifolds. We discuss the lower volume bounds on even-dimensional negatively curved rank 1 symmetric spaces. (This is joint work with P. Gilkey)

June 20, Tuesday

A condition for a Riemannian manifold to be globally homogeneous -Part 2-

10:00 am

Joseph A. Wolf
University of California, Berkeley, USA

Consider a connected homogeneous Riemannian manifold (M, ds^2) and a Riemannian covering $(M, ds^2) \to \Gamma\backslash(M, ds^2)$. If $\Gamma\backslash(M, ds^2)$ is homogeneous then every $\gamma \in \Gamma$ is an isometry of constant displacement. The Homogeneity Conjecture suggests the converse: if every $\gamma \in \Gamma$ is an isometry of constant displacement on (M, ds^2) then $\Gamma\backslash(M, ds^2)$ is homogeneous. We survey the cases in which the Homogeneity Conjecture has been verified, plus some new results, and suggest some related open problems.

Second hour: Manifolds of positive Euler characteristic, group manifolds, manifolds of positive curvature, noncompact manifolds, open problems

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Integral transformations and Two Radius Theorems in noncompact harmonic manifolds

11:00 am

Norbert Peyerimhoff
Durham University, England

In this talk we introduce various integral transformations like the Abel transform, the dual Abel transform and the spherical Fourier transform and discuss relations between them. We prove that functions satisfying the mean value property for two generic radii and arbitrary midpoints must be harmonic and that functions whose averages over two generic radii and arbitrary midpoints vanish must be identically zero (2 Radius Theorems), generalising earlier results by Berenstein/Zalcman for rank one symmetric spaces. If time permits, we also discuss a Paley-Wiener Theorem for radial distributions and an application to the heat kernel.

2:00 pm

Geodesic orbit pseudo Riemannian nilmanifolds

Yuri Nikolayevsky
La Trobe University, Australia

We know that in the Riemannian case, (i) for every homogeneous space, there is a reductive decomposition at the level of Lie algebras, (ii) the isometry group of a simply connected nilmanifold is the semidirect product of isometric automorphisms and translations (Wolf/Wilson), and (iii) geodesic orbit nilmanifolds are necessarily two-step nilpotent or abelian (Gordon). Neither of this is true in pseudo-Riemannian signature. However, it turns out that in low signature, some results may still be "rescued". This is a joint work (which is partially still in progress) with Joe Wolf, Zhiqi Chen and Shaoxiang Zhang.

3:00 pm

Asymptotically harmonic manifolds in 3

Hemangi Shah
Harish-Chandra Research Institute, India

We will show that an asymptotically harmonic manifold in dimension 3 is a symmetric space. Thus we obtain the complete classification of asymptotically harmonic manifolds in dimension 3.

4:30 pm

Second order Einstein deformations

Paul-Andi Nagy
Institute for Basic Science, Korea

We study the integrability to second order of Einstein metrics on compact Riemannian and in particular on Kähler manifolds. We find a new way of expressing the necessary and sufficient condition for integrability to second order, which also gives a very clear and compact way of writing the Koiso obstruction. As an application we consider the Kähler case, where the condition can be further simplified and in complex dimension 3 turns out to be purely algebraic. For Kähler-Einstein metrics of negative scalar curvature we show that the Einstein deformation theory is unobstructed to second order. This is joint work with Uwe Semmelmann.

June 21, Wednesday

Sphere-like isoparametric hypersurfaces in Damek-Ricci spaces 10:00 am

Balázs Csikós

Eötvös Loránd University, Hungary

Locally harmonic spaces are Riemannian manifolds in which small geodesic spheres are isoparametric hypersurfaces, i.e., hypersurfaces whose nearby parallel hypersurfaces are of constant mean curvature. Flat and rank one symmetric spaces are examples of harmonic spaces. Damek-Ricci spaces are non-compact harmonic spaces, most of which are non-symmetric. Taking the limit of an "inflating" sphere through a point P in a Damek-Ricci space as the center of the sphere runs out to infinity along a geodesic half-line γ starting from P, we get a horosphere. Similarly to spheres, horospheres are also isoparametric hypersurfaces. In this talk, we define and study the geometric properties of the sphere-like hypersurfaces obtained by "overinflating the horospheres" by pushing the center of the sphere beyond the point at infinity of γ along a virtual prolongation of γ. We show for example, that these hypersurfaces are isoparametric hypersurfaces that can be obtained as tubes about a minimal submanifold of lower dimension.

The Helgason Fourier Transform on non-compact harmonic manifolds 11:00 am

Gerhard Knieper
(joint work with Kingshook Biswas and Norbert Peyerimhoff)

Ruhr-Universität Bochum, Germany

In this talk we discuss a (non-spherical) Fourier transform for non-compact harmonic Gromov hyperbolic manifolds. This is an extension of a Fourier transform for rank 1 symmetric spaces of non-compact type due to Helgason which in turn generalizes the classical Fourier transform in $\mathbb{R}^\kappa$. We will prove an Inversion formula and a Plancherel theorem which for rank 1 symmetric spaces of non-compact type has been established by Helgason. However, unlike in the case of symmetric spaces tools from harmonic analysis on Lie groups are not available. Instead our approach is based on methods from Riemannian geometry and and tools from the theory of hypergroups.

2:00 pm

Damek-Ricci spaces as hypersurfaces in symmetric spaces

Norbert Peyerimhoff
(joint work with Gerhard Knieper and John R Parker)
Durham University, England

A surprising discovery due to Damek-Ricci was that one-dimensional solvable extensions of particular 2-step nilpotent Lie groups with left-invariant metrics are harmonic. In this talk, we consider minimal hypersurfaces of particular irreducible rank 2 symmetric space of noncompact type. It turns out that, in certain cases, they are harmonic spaces. Via this process, we recover certain Damek-Ricci spaces like the complex hyperbolic plane and non-symmetric 7- and 13-dimensional Damek-Ricci spaces. We consider this as an intriguing fact and we wonder whether there is a more general principle behind this observation. So this talk is a bit open-ended.

3:00 pm

Asymptotic harmonicity and the generalization of the Hopf conjecture

Hemangi Shah
Harish-Chandra Research Institute, India

We will prove the Generalization of the Hopf conjecture in the settings of harmonic and homogeneous asymptotically harmonic manifolds satisfying Euclid's Parallel Postulate. A long-standing open conjecture in the theory of simply connected, complete Riemannian manifold without conjugate points is the Generalization of the Hopf Conjecture. The Conjecture states that, if (M^n, g) is a simply connected, complete Riemannian manifold without conjugate points satisfying Euclid's parallel postulate, then M is flat. The Conjecture is established by affirming the property that, (M, g) satisfies Euclid's parallel postulate if and only if the Busemann function B_v for a unit tangent vector v of M satisfies $B_v + B_{-v} = 0$ in all directions.

4:30 pm

The shifted wave equation on non flat harmonic manifolds

Oliver Brammen
Ruhr-Universität Bochum, Germany

Let (X, g) be a non-compact simply connected harmonic manifold with mean curvature of the horospheres $2\rho > 0$. The main object of this talk is to present a characterization of smooth solution of the shifted wave equation $\frac{\partial^2}{\partial t^2}\phi = (\Delta + \rho^2)\phi$ on X with compactly supported initial conditions, via the inverse dual Abel transform. This generalizes the homogeneous case.

Integrals on the intersections of two horospheres in asymptotically harmonic spaces

5:00 pm

Sinhwi Kim

Sungkyunkwan University, Korea

It is shown that, in an asymptotically harmonic manifold, a volume-preserving mapping can be constructed for each pair of two distinct points. If the asymptotically harmonic manifold satisfies the visibility condition, it is also shown that the mappings which preserve distances in some directions can be constructed. We use such mappings to prove that some integrals on the intersection of two horospheres are independent of the differences between the values of the corresponding Busemann functions. (This is joint work with J. H. Park)

June 22, Thursday

New developments on Einstein manifolds with symmetry -Part 2-

10:00 am

Ramiro Lafuente

University of Queensland, Australia

I will discuss the recent resolution of the 1975 Alekseevskii conjecture on homogeneous Einstein manifolds, in joint work with Christoph Böhm. I will also describe new results on the structure and rigidity of Einstein manifolds with a cocompact symmetry group, including the fact that a negative Einstein manifold with a cocompact unimodular group of isometries must split isometrically as a product of a compact Einstein manifold and a symmetric space of non-compact type.

Einstein hypersurfaces in irreducible symmetric spaces

11:00 am

Yuri Nikolayevsky

La Trobe University, Australia

In this talk, I will present the results of the joint paper of Jeong Hyeong Park and myself in which we give a classification of Einstein hypersurfaces in irreducible symmetric spaces. The main theorem states that there are three classes of such hypersurfaces, belonging to three very different "geometries": homogeneous geometry, Legendrian geometry and affine geometry. I will give a brief introduction into these three geometries and explain how they fit together in our classification.
